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MOTION OF A SYSTEM OF SEISMIC SOURCES OVER ICE ON A BODY OF WATER UNDER THE ACTION OF A PULSE

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The vertical motion of a system of two identical seismic sources and a spring truck tractor on ice under the action of a shock pulse from the seismic sources is studied to estimate the strength of ice. It is shown that during the pulse time, the interaction of the masses of the seismic sources and the tractor is small and the compressibility effect of the liquid can be ignored. Calculations show that for the seismic sources, the dynamic load far exceeds the static load and for the tractor, the static load is maximal.

Key words: *pulse load, floating ice plate, structural damping, compressible and incompressible liquids.*

Geophysical work on the ice sheet of bodies of water is needed for oil prospecting in northern regions. The effect of a point shock pulse on a floating ice sheet was considered in [1–4]. In the present paper, the vertical motion of a system of masses (one of which is spring mounted) on ice under the action of a shock pulse is studied with allowance for the bearing surface area. The object of the study is to determine the mass accelerations of the seismic sources and tractor in order to assess the safety of operation of the geophysical equipment on a floating ice sheet.

Formulation of the Problem. The geophysical equipment consists of two identical seismic sources, a spring tractor at identical distances from them, and a head tractor, which is assumed to be very remote and is therefore not considered (Fig. 1). The masses of the seismic sources and tractor are equal to 4000 and 8000 kg, respectively. Each seismic source is mounted on two skids 2 m long and 0.5 m wide, and the tractor is on skids 6.5 m long and 1 m wide. The seismic sources operate synchronously. The pulse time is $t_0 = 13.6$ msec, after which the oscillations of the spring-mounted tractor damp gradually. The acceleration of the seismic sources during the pulse time is known from experiments on ground. The rigidity of the springs can be determined taking into account that the deflection of the tractor under gravity is 0.2 m. It is assumed that the liquid is ideal and infinitely deep and that the ice sheet is homogeneous and have constant thickness and infinite extension. The problem is solved using the theory of thin plates with structural damping taken into account [5]. The problem is solved for the cases of both compressible and incompressible liquids. The vertical motion of the system of sources and tractor on ice is defined by the equations

$$\rho_1 h \frac{\partial^2 w}{\partial t^2} + \Big(1 + \alpha \frac{\partial}{\partial t}\Big) D \Delta w = p(x, y, t) + F(x, y, t)$$

$$- \frac{M}{S} \sum_{j=1}^{2} A_j(x, y) \ddot{Z}_j(t) + k_b (Z_b - W_b) \frac{A_b(x, y)}{S_b},$$

$$F(x, y, t) = -\frac{M}{S} \ddot{Z}_0(t) \sum_{j=1}^{2} A_j(x, y), \qquad M_b \ddot{Z}_b + k_b (Z_b - W_b) = 0, \tag{1}$$

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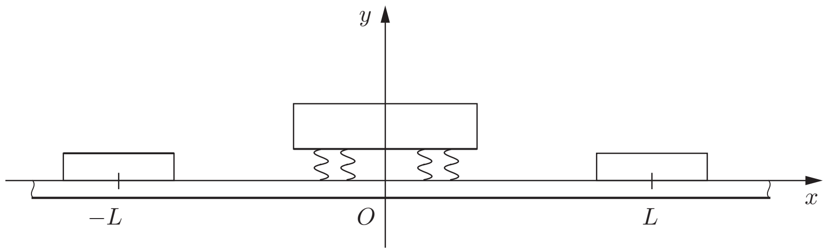

Fig. 1. Arrangement of the geophysical equipment.

$$Z_j(t) = W_j(t), \qquad D = Eh^3/[12(1-\nu^2)],$$

$$p = -\rho(\varphi_t + gw), \qquad \Delta\varphi - \varphi_{tt}/c_0^2 = 0,$$

$$\varphi_z = w_t \quad (z=0), \qquad \nabla\varphi \to 0, \quad z \to -\infty.$$

Here $w(x,y,t)$ are the displacements of the ice plate, α is the structural damping coefficient, D and h are the flexural rigidity and thickness of the plate, E and ν are Young's modulus and Poisson's constant for ice, ρ_1 and ρ are the densities of ice and water, respectively, p is the hydrodynamic pressure, φ is the liquid velocity potential, c_0 is the sound velocity in water, g is the acceleration of gravity, $F(x,y,t)$ is the effect of the seismic sources, M and M_b are the masses of the seismic sources and tractor, S and S_b are the areas of the skids of the seismic sources and tractor, k_b is the rigidity of the tractor springs, $Z_j(t)$ is a function of displacements of the seismic sources, $Z_b(t)$ is a function of s displacements of the tractor, $W_j(t)$ and $W_b(t)$ are functions of ice displacements under the seismic sources and tractor, (x_j, y_j) and (x_b, y_b) are the horizontal coordinates of the centers of gravity of the seismic sources and tractor, $\ddot{Z}_0(t)$ is the specified acceleration of the seismic sources during the pulse time, and $A_j(x,y)$ and $A_b(x,y)$ are functions of the surface area of the sources and tractor:

$$A_j(x,y) = \begin{cases} 1, & (x,y) \in D_j, \\ 0, & (x,y) \notin D_j, \end{cases} \qquad A_b(x,y) = \begin{cases} 1, & (x,y) \in D_b, \\ 0, & (x,y) \notin D_b; \end{cases}$$

D_j and D_b are the areas occupied by the skids of the seismic sources and tractor respectively. The displacements of the seismic sources and ice at their centers of gravity are set equal (the displacement of the seismic sources relative to the ice surface is ignored):

$$W_j(t) = Z_j(t) = w(x_j, y_j, t), \qquad W_b(t) = w(x_b, y_b, t). \tag{2}$$

Zero initial conditions are specified, and at $t < 0$, the system is at rest. Once the functions $Z_j(t)$, $Z_b(t)$, and $W_b(t)$ are determined, the displacement of the ice plate, stresses, and shear forces can be found from Eqs. (1).

System of Integral Equations. The contact surface area between the skids and ice is important in determining the motion of the seismic sources and tractor, and the shape of the contact area will be considered circular. After the motion of the masses of the seismic sources and tractor is determined, the stress on ice can be found taking into account the shape of the skids. Then, Eq. (1) becomes

$$\rho_1 h \frac{\partial^2 w}{\partial t^2} + \Big(1 + \alpha \frac{\partial}{\partial t}\Big) D\Delta w = p(x,y,t) - \frac{M}{S}\, \ddot{Z}_0(t) \sum_{j=1}^{2} \tilde{A}_j(x,y)$$

$$- \frac{M}{S} \sum_{j=1}^{2} \tilde{A}_j(x,y) \ddot{Z}_j(t) + k_b(Z_b - W_b)\, \frac{\tilde{A}_b(x,y)}{S_b}, \tag{3}$$

where $\tilde{A}_j(x,y)$ and $\tilde{A}_b(x,y)$ are circles of area S and S_b and radius R and R_b with centers at the points (x_j, y_j) and (x_b, y_b).

To solve the problem, we use the Fourier transform over the space coordinates x and y and the Laplace transform in time:

$$\hat{w}^F(\xi,\eta,s)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{0}^{\infty}\mathrm{e}^{i(\xi x+\eta y)}\,\mathrm{e}^{st}\,w(x,y,t)\,dx\,dy\,dt,$$

$$\hat{\varphi}^F(\xi,\eta,z,s)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{0}^{\infty}\mathrm{e}^{i(\xi x+\eta y)}\,\mathrm{e}^{st}\,\varphi(x,y,z,t)\,dx\,dy\,dt,$$

$$\hat{p}^F(\xi,\eta,z,s)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{0}^{\infty}\mathrm{e}^{i(\xi x+\eta y)}\,\mathrm{e}^{st}\,p(x,y,z,t)\,dx\,dy\,dt.$$

Here $\hat{w}^F(\xi,\eta,s)$, $\hat{\varphi}^F(\xi,\eta,z,s)$, and $\hat{p}^F(\xi,\eta,z,s)$ are the images of the functions $w(x,y,t)$, $\varphi(x,y,z,t)$, and $p(x,y,z,t)$.

From the equation of motion for the liquid, we obtain

$$\frac{\partial^2\hat{\varphi}^F}{\partial z^2}-\Big(\xi^2+\eta^2+\frac{s^2}{c_0^2}\Big)\hat{\varphi}^F=0.$$

The solution of this equation subject to the condition that it damps as $z\to-\infty$ has the form

$$\hat{\varphi}^F(\xi,\eta,z,s)=C(\xi,\eta,s)\,\mathrm{e}^{\sigma z},\qquad \sigma=\sqrt{\xi^2+\eta^2+s^2/c_0^2}.$$

From the boundary conditions for $z=0$, we have

$$s\hat{w}^F=\sigma\hat{\varphi}^F(\xi,\eta,0,t),\qquad \hat{\varphi}^F(\xi,\eta,0,s)=s\hat{w}^F/\sigma.$$

Hence,

$$p^F=-\rho(s^2/\sigma+g)\hat{w}^F.$$

The equation of motion for the plate (3) becomes

$$\rho_1hs^2\hat{w}^F+(1+\alpha s)D(\xi^2+\eta^2)^2\hat{w}^F=-\rho\Big(\frac{s^2\hat{w}^F}{\sigma}+g\hat{w}^F\Big)$$

$$-\frac{MJ_1(R\gamma)}{\pi\gamma R}\sum_{j=1}^{2}\mathrm{e}^{i(\xi x_j+\eta y_j)}[s^2\hat{Z}_j(s)+\hat{\ddot{Z}}_0(s)]-\frac{M_bJ_1(R_b\gamma)}{\pi\gamma R_b}\,\mathrm{e}^{i(\xi x_b+\eta y_b)}\,s^2\hat{Z}_b(s).\tag{4}$$

Here $\gamma=\sqrt{\xi^2+\eta^2}$, $\xi=\gamma\cos\theta$, $\eta=\gamma\sin\theta$, γ and θ are cylindrical coordinates in the plane (ξ,η), and J_1 is a Bessel function. Equation (4) can be written as

$$s^2m(\gamma)\hat{w}^F+sd(\gamma)\hat{w}^F+c(\gamma)\hat{w}^F=f(\gamma,\theta,s),\tag{5}$$

where

$$m(\gamma)=\rho_1h+\rho/\sigma,\qquad d(\gamma)=\alpha D\gamma^4,\qquad c(\gamma)=D\gamma^4+\rho g,$$

$$f(\gamma,\theta,s)=-\frac{MJ_1(R\gamma)}{\pi\gamma R}\sum_{j=1}^{2}\mathrm{e}^{i\gamma(x_j\cos\theta+y_j\sin\theta)}[s^2\hat{Z}_j(s)+\hat{\ddot{Z}}_0(s)]$$

$$-\frac{M_bJ_1(R_b\gamma)}{\pi\gamma R_b}\,s^2\hat{Z}_b\,\mathrm{e}^{i\gamma(x_b\cos\theta+y_b\sin\theta)}.$$

The solution of Eq. (5) has the form

$$\hat{w}^F(\xi,\eta,s)=f(\gamma,\theta,s)/K(\gamma,s),$$

$$K(\gamma,s)=D(1+s\alpha)\gamma^4+\rho g+\rho_1hs^2+\rho s^2/\sigma,\qquad \sigma=\sqrt{\gamma^2+s^2/c_0^2}$$

[$K(\gamma,s)$ is the dispersion function for flexural-gravity waves in the floating elastic plate]. In the case of an incompressible liquid,

$$K(\gamma, s) = D(1 + s\alpha)\gamma^4 + \rho g + \rho_1 h s^2 + \rho s^2/\gamma.$$

Using the inverse Fourier transform, we obtain

$$\hat{w}(x, y, s) = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} \mathrm{e}^{-i\gamma(x\cos\theta + y\sin\theta)}\, \hat{w}^F(\xi, \eta, s)\gamma\, d\gamma\, d\theta.$$

For the system of the sources and the tractor between them, arranged on the same line, we choose a coordinate system with origin at the middle of the tractor and the Ox axis directed along the direction of motion of the system (see Fig. 1). Then, $(-L, 0)$ and $(L, 0)$ are the coordinates of the seismic sources, where L is the distance between the centers of the seismic sources and tractor. By virtue of symmetry, the displacements of the seismic sources are identical: $Z_1(t) \equiv Z_2(t) \equiv Z(t)$. Then, with allowance for (2), the motion of the seismic sources and tractor is described by the system of equations

$$\begin{gathered}
\hat{Z}(s) = -\frac{M}{\pi R} \int_0^\infty J_1(R\gamma)[1 + J_0(2\gamma L)][s^2\hat{Z}(s) + \hat{\ddot{Z}}_0(s)]\, \frac{d\gamma}{K(\gamma, s)} - \frac{M_b s^2}{\pi R_b} \int_0^\infty J_0(\gamma L) J_1(R_b\gamma)\hat{Z}_b(s)\, \frac{d\gamma}{K(\gamma, s)}, \\
\hat{W}_b(s) = -\frac{2M}{\pi R} \int_0^\infty J_0(\gamma L) J_1(R\gamma)[s^2\hat{Z}(s) + \hat{\ddot{Z}}_0(s)]\, \frac{d\gamma}{K(\gamma, s)} - \frac{M_b s^2}{\pi R_b} \int_0^\infty J_1(R_b\gamma)\hat{Z}_b(s)\, \frac{d\gamma}{K(\gamma, s)}, \\
M_b s^2 \hat{Z}_b + k_b(Z_b - W_b) = 0.
\end{gathered} \qquad (6)$$

Solution for One Seismic Source. From the equation for one seismic source

$$\hat{Z}(1 + Ms^2 B_0(s)) = -MB_0(s)\hat{\ddot{Z}}_0 \qquad \Big(B_0(s) = \frac{1}{\pi R} \int_0^\infty \frac{J_1(R\gamma)\, d\gamma}{K(\gamma, s)}\Big),$$

we obtain

$$\hat{Z}(s) = -\hat{\ddot{Z}}_0\, \frac{MB_0(s)}{1 + Ms^2 B_0(s)}. \qquad (7)$$

The function $Z(t)$ describing the motion of the seismic source in time can be found using an inverse Laplace transform. However, to determine the deflection of the ice plate from Eq. (1) and the stress in it, it is necessary to find the acceleration of the seismic source $\ddot{Z}(t)$. For the image of the acceleration, from (7) we obtain

$$\hat{\ddot{Z}}(s) = -\hat{\ddot{Z}}_0 F_2(s), \qquad F_2(s) = \frac{Ms^2 B_0(s)}{1 + Ms^2 B_0(s)}. \qquad (8)$$

Let us find the asymptotic form of the expression $s^2 B_0(s)$ as $|s| \to \infty$. For this, we introduce the characteristic length l related to the elastic forces, and the dimensionless variables

$$l = \Big(\frac{D}{\rho g}\Big)^{1/4}, \quad \zeta = \gamma l, \quad \bar{R} = \frac{R}{l}, \quad \bar{R}_b = \frac{R_b}{l}, \quad \bar{L} = \frac{L}{l}, \quad \bar{c}_0 = \frac{c_0 t_0}{l}.$$

Then, the expression for $s^2 B_0(s)$ is written as

$$s^2 B_0(s) = \frac{1}{\pi \bar{R}\sqrt{\rho g D}} \int_0^\infty \frac{\chi(\zeta, s) J_1(\bar{R}\zeta)\, d\zeta}{\chi(\zeta, s)\zeta^4(1/s^2 + \alpha/s) + \chi(\zeta, s)(\lambda + 1/s^2) + \mu},$$

$$\lambda = \rho_1 h/(\rho g), \quad \mu = l/g, \quad \chi(\zeta, s) = \sqrt{\zeta^2 + s^2/\bar{c}_0^2}.$$

As $|s| \to \infty$, we have $s^2 B_0(s) \to C_{0s}$, where $C_{0s} = 1/(\pi\lambda\bar{R}^2\sqrt{\rho g D})$. In the case of an incompressible liquid, we have

$$C_{0s} = \frac{1}{\pi \bar{R}\sqrt{\rho g D}} \int_0^\infty \frac{J_1(\bar{R}\zeta)\zeta\, d\zeta}{\zeta\lambda + \mu}.$$

For the inversion of the Laplace transform, we write the function $F_2(s)$ as

$$F_2(s) = sF_1(s), \qquad F_1(s) = \frac{MsB_0(s)}{1 + Ms^2B_0(s)}.$$

The function $F_1(s) = C/s + O(|s|^{-2})$ as $|s| \to \infty$. Here $C = MC_{0s}/(1 + MC_{0s})$. In the case of an incompressible liquid, $C = 0.6383$, and for a compressible liquid, $C = 0.8163$. We denote the original of the function $F_1(s)$ by $Q(t)$. Then, the solution of Eq. (8) has the form

$$\ddot{Z}(t) = -\frac{d}{dt}\int\limits_0^t Q(t-\tau)\ddot{Z}_0(\tau)\,d\tau. \tag{9}$$

Using the inverse Laplace transform, we obtain

$$Q(t) = \frac{1}{\pi}\int\limits_0^\infty \mathrm{Re}\,[\mathrm{e}^{i\omega t}\,F_1(i\omega)]\,d\omega. \tag{10}$$

Inversion of the Laplace transform was performed numerically. The compressibility effect for flexural-gravity waves is manifested only at high frequencies [1]; therefore, for frequencies smaller than 1 sec^{-1}, the compressibility was ignored. For small values of ω, the dispersion function has a zero located close to the imaginary axis and numerical calculation of the integral $B_0(i\omega)$ is difficult. Writing the function $1/K(\gamma, s)$ as the sum of common fractions

$$\frac{1}{K(\gamma, s)} = \sum_{k=0}^{4}\frac{A_k}{\gamma - \gamma_k}$$

(γ_k are roots of the dispersion relation) and using the table integral

$$\int\limits_0^\infty \frac{xJ_1(cx)\,dx}{x+z} = -\frac{\pi z}{2}\,[H_{-1}(cz) - Y_{-1}(cz)],$$

we obtain

$$B_0(s) = \sum_{k=0}^{4}\frac{A_k\gamma_k}{2\bar{R}\sqrt{\rho g D}}\,[H_{-1}(-\bar{R}\gamma_k) - Y_{-1}(-\bar{R}\gamma_k)]$$

(H_{-1} and Y_{-1} are Struve and Bessel functions, respectively).

The calculations were performed for the following parameter values of ice and water: $\rho_1 = 900$ kg/m^3, $\rho = 1000$ kg/m^3, $E = 5\cdot 10^9$ N/m^2, $\nu = 0.3$, $h = 0.5$ m, $\alpha = 0.69$ sec, and $L = 10$ m. Previously, it has been shown [6, 7] that the viscoelastic model of ice provides the best fit to experimental data for $\alpha = (0.69 \pm 0.067)$ sec. The time dependence of the specified acceleration of the seismic source on ground under the action of the pulse is shown in Fig. 2 by a solid curve.

According to the calculations, the asymptotic form of the integral $B_0(i\omega)$ is valid with adequate accuracy (a difference is observed in the fourth decimal place) for high frequencies ($\omega > \omega_*$); for an incompressible liquid, $\omega_* = 2\cdot 10^8$ sec^{-1}, and for a compressible liquid, $\omega_* = 2\cdot 10^9$ sec^{-1}. The interval $[0, \omega_*]$ was divided into segments on which the real and imaginary parts of the function $F_1(i\omega)$ were approximated by polynomials for $\omega < 1$ and by polynomials in ascending and descending powers of order not higher than two for $\omega > 1$. The approximation error is small enough (a difference was observed in the fourth decimal place). For $\omega > \omega_*$, the function $F_1(i\omega)$ was replaced by the asymptotic relation $-iC/\omega$. The integral (10) was evaluated analytically using this approximation.

From the calculations, it follows that the compressibility effect on the function $Q(t)$ is manifested only for small times ($t < t_*$) and $t_* \ll t_0$; therefore, the compressibility effect of the liquid on the acceleration of the seismic source is small. In Fig. 3, the time dependence of the acceleration of a single seismic source on ice is shown by a dashed curve for the case of a compressible liquid and by a dotted curve in the case of an incompressible liquid. From Fig. 3, it is evident that the curves almost coincide; therefore, for the system of two seismic sources and a tractor, the calculations were performed for the case of an incompressible liquid.

Solution for a System of Two Seismic Sources and a Tractor. Using the Laplace transform in dimensionless variables, from system (6) we obtain the system of equations

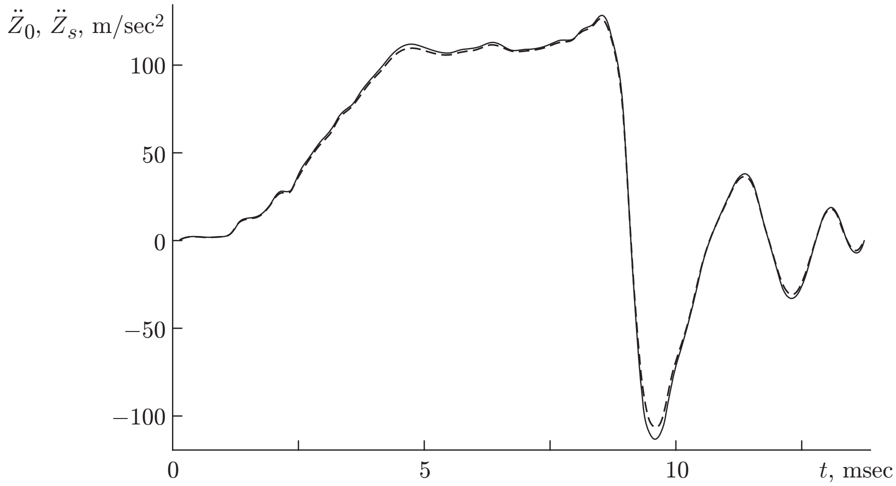

Fig. 2. Acceleration of the seismic source versus time: the solid curve refers to a single seismic source on ground; the dashed curve refers to the case where one of the seismic sources of the system is on ice.

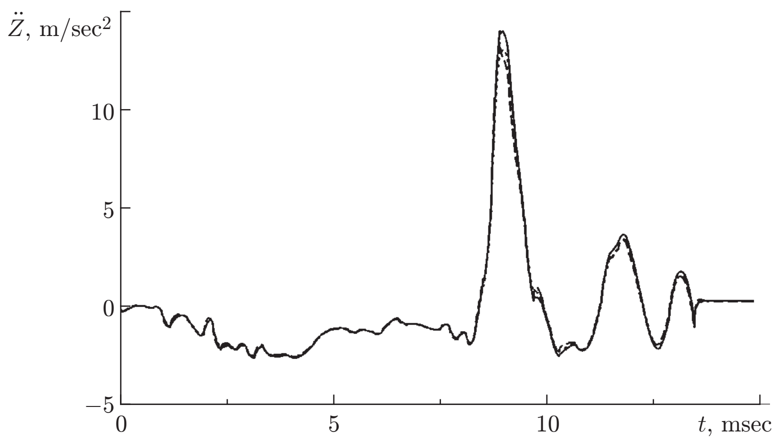

Fig. 3. Acceleration of seismic sources on ice versus time: the solid curve refers to the system of two seismic sources and a tractor; the dashed curve refers to a single seismic source for the case of a compressible liquid; and the dotted curve refers to a single seismic source for the case of an incompressible liquid.

$$A\begin{pmatrix} \hat{Z} \\ \hat{Z}_b \end{pmatrix} = -\hat{\ddot{Z}}_0 M \begin{pmatrix} B_{0s} + B_2 \\ 2B_{1s} \end{pmatrix},$$

where

$$A = \begin{pmatrix} 1 + Ms^2(B_{0s} + B_2) & M_b s^2 B_{1b} \\ 2Ms^2 B_{1s} & 1 + M_b s^2/k_b + M_b s^2 B_{0b} \end{pmatrix},$$

$$B_{0s}(s) = I_0(\bar{R}, s), \quad B_{0b}(s) = I_0(\bar{R}_b, s), \quad B_{1s}(s) = I_1(\bar{R}, s), \quad B_{1b}(s) = I_1(\bar{R}_b, s), \quad B_2(s) = I_2(\bar{R}, s),$$

$$I_0(R, s) = \frac{1}{\pi R\sqrt{\rho g D}} \int\limits_0^\infty \frac{J_1(R\zeta)\, d\zeta}{\tilde{K}(\zeta, s)}, \qquad I_j(R, s) = \frac{1}{\pi R\sqrt{\rho g D}} \int\limits_0^\infty \frac{J_1(R\zeta) J_0(j\bar{L}\zeta)\, d\zeta}{\tilde{K}(\zeta, s)},$$

$$\tilde{K}(\zeta, s) = \zeta^4(1 + \alpha s) + 1 + \lambda s^2 + \mu s^2/\zeta.$$

For the images of the accelerations, this system is written as

$$A\begin{pmatrix}\hat{\ddot{Z}}\\ \hat{\ddot{Z}}_b\end{pmatrix} = -\hat{\ddot{Z}}_0 M s^2 \begin{pmatrix} B_{0s}+B_2 \\ 2B_{1s}\end{pmatrix}. \tag{11}$$

Solving system (11), we find the images of the accelerations of the seismic sources and tractor, and then, using the inverse Laplace transform, we find the time dependence of the accelerations.

The solution of system (11) has the form

$$\hat{\ddot{Z}}(s) = -\hat{\ddot{Z}}_0 G(s), \qquad \hat{\ddot{Z}}_b(s) = -\hat{\ddot{Z}}_0 G_b(s),$$

$$G(s) = \frac{Ms^2(B_{0s}+B_2)(1+M_b s^2/k_b + M_b s^2 B_{0b}) - 2MM_b s^4 B_{1s}B_{1b}}{[1+Ms^2(B_{0s}+B_2)](1+M_b s^2/k_b + M_b s^2 B_{0b}) - 2MM_b s^4 B_{1s}B_{1b}},$$

$$G_b(s) = \frac{2Ms^2 B_{1s}}{[1+Ms^2(B_{0s}+B_2)](1+M_b s^2/k_b + M_b s^2 B_{0b}) - 2MM_b s^4 B_{1s}B_{1b}}.$$

We find the asymptotic forms of the integrals as $|s| \to \infty$: $s^2 B_{1s} \to C_{1s}$, $s^2 B_2 \to C_2$, and $s^2 B_{1b} \to C_{1b}$, where

$$C_{1s} = \frac{1}{\pi \bar{R}\sqrt{\rho g D}} \int_0^\infty \frac{J_1(\bar{R}\zeta)J_0(\bar{L}\zeta)\zeta\, d\zeta}{\zeta\lambda + \mu}, \qquad C_2 = \frac{1}{\pi \bar{R}\sqrt{\rho g D}} \int_0^\infty \frac{J_1(\bar{R}\zeta)J_0(2\bar{L}\zeta)\zeta\, d\zeta}{\zeta\lambda + \mu},$$

$$C_{1b} = \frac{1}{\pi \bar{R}_b\sqrt{\rho g D}} \int_0^\infty \frac{J_1(\bar{R}_b\zeta)J_0(\bar{L}\zeta)\zeta\, d\zeta}{\zeta\lambda + \mu}.$$

Finally, we obtain $G(s) \to C_1$ and $C_1 = 0.6382$, i.e., the constant is almost the same as for a single seismic source, and $s^2 G_b(s) \to C_b$ and $C_b = 0.022\,36$.

The acceleration of the seismic sources for the system is found, as in the case of a single seismic source, from formulas (9), (10), and the acceleration of the tractor is determined from the expressions

$$\ddot{Z}_b(t) = -\int_0^t Q_b(t-\tau)\ddot{Z}_0(\tau)\, d\tau, \qquad Q_b(t) = \frac{1}{\pi}\int_0^\infty \mathrm{Re}\,[\mathrm{e}^{i\omega t}\, G_b(i\omega)]\, d\omega.$$

We note that system (11) has poles in the left half-plane near the imaginary axis and the values $\pm i\omega_0$, where $\omega_0 = \sqrt{50}$ is the eigenfrequency of the spring-mounted tractor. The presence of structural damping and spatial energy dissipation shifts the poles to the left half-plane. For the system considered, the eigenfrequency is found using the argument principle $\omega_* = 7.011\,49 + 0.013\,097i$. In the neighborhood of the eigenfrequency, the functions $G(i\omega)$ and $G_b(i\omega)$ undergo sudden changes. The inversion of the Laplace transform was performed in the same way as for a single seismic source, with the only difference that for the approximation near the eigenfrequency, instead of negative powers, we used functions of the form

$$\frac{\omega}{(\omega-\omega_r)^2 + \omega_i^2}, \qquad \frac{1}{(\omega-\omega_r)^2 + \omega_i^2},$$

where ω_r and ω_i are the real and imaginary parts of the eigenfrequency ω_*.

To estimate the strength of ice, it is necessary to know the maximum forces acting on the ice sheet. In the neighborhood of the seismic source, the maximum forces are observed during the pulse time. The natural oscillations of the seismic sources are much smaller than the oscillations of the tractor. In Fig. 3, the solid curve shows the time dependence of the acceleration of the seismic sources for the system (the dashed and dotted curves refer to a single seismic source for the cases of a compressible and an incompressible liquid, respectively). As is evident from Fig. 3, all curves almost coincide. Figure 2 gives the time dependences of the specified acceleration of a single seismic source on ground (solid curve) and the resultant acceleration $\ddot{Z}_s(t) = \ddot{Z}_0(t) + \ddot{Z}(t)$ of a seismic source on ice under the action of the system (dashed curve). From Figs. 2 and 3, it follows that for the calculation of the maximum dynamic stresses in ice, the interaction of the seismic sources and tractor can be ignored at distances between them of the order of 10 m.

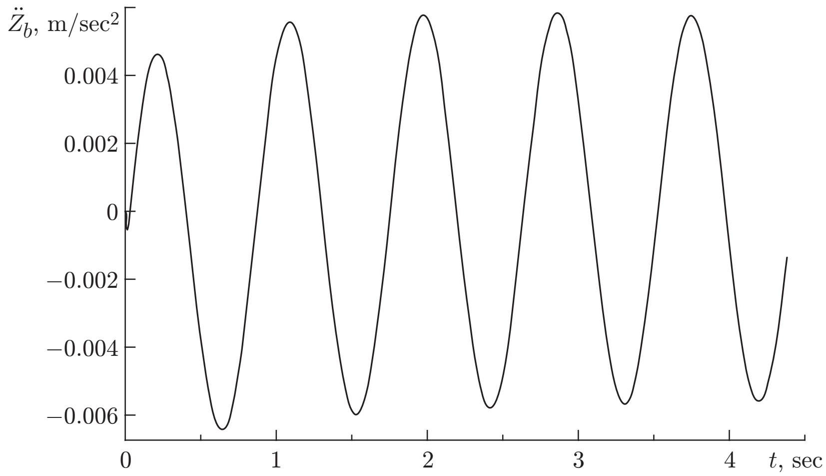

Fig. 4. Acceleration of the tractor versus time.

Figure 4 gives a curve of the acceleration of the tractor versus time. It is evident that the pulse of the seismic sources excites natural oscillations of the spring-mounted tractor. These oscillations damp weakly enough but their amplitudes are small.

The results show that for the seismic sources, the dynamic loads far exceed the static loads, and for the tractor, the static loads are the maximal ones.

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